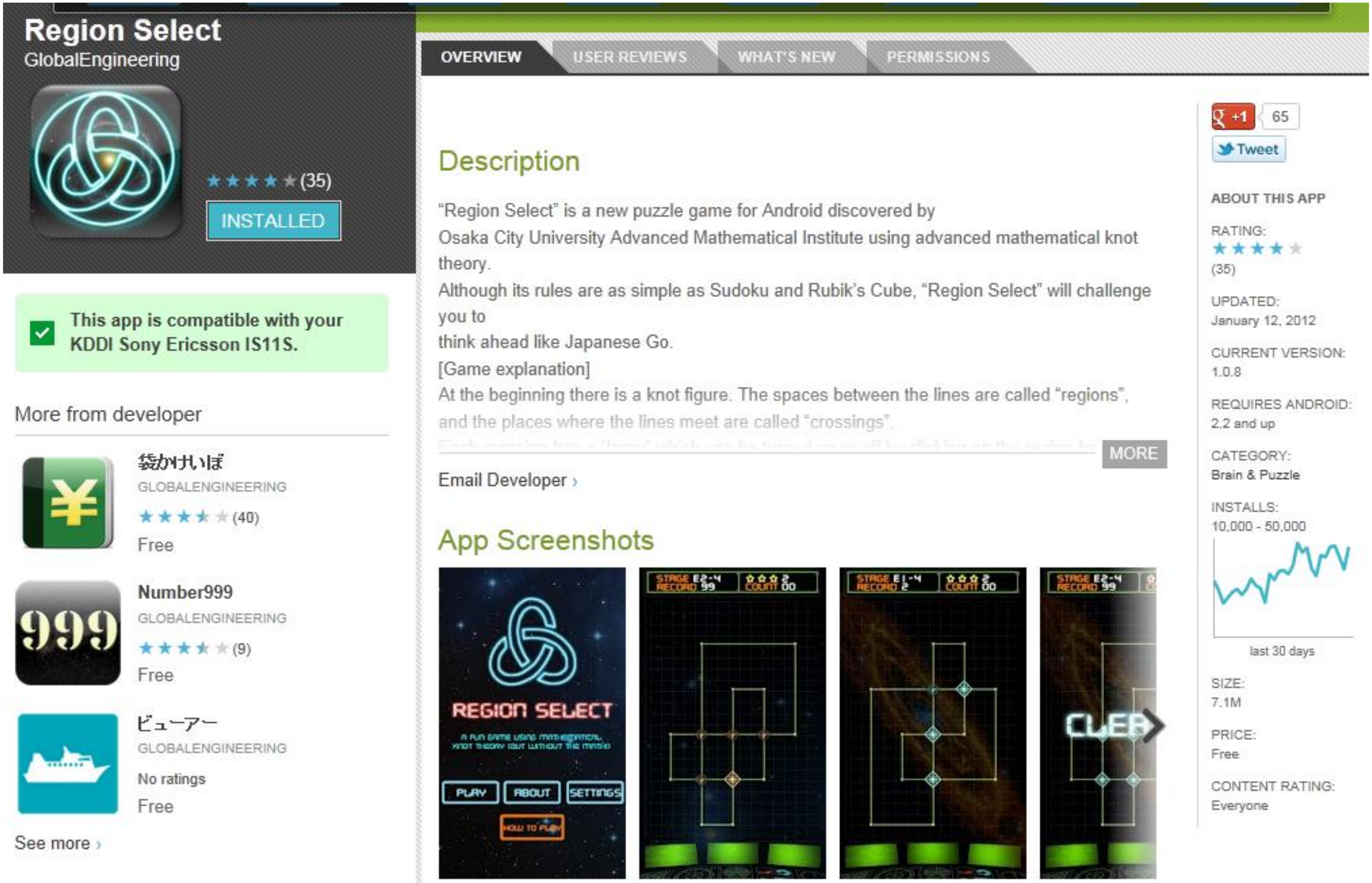
Games & a Switch using Knot Theory

w/ Akio Kawauchi & Kengo Kishimoto

by Ayaka Shimizu (Hiroshima University/OCAMI)

Mathematical Software and Free Documents XV

September 17, 2012


Region Select
GlobalEngineering
(35)
INSTALLED
This app is compatible with your KDDI Sony Ericsson IS11S.
More from developer
袋かけいぼ
GLOBALENGINEERING
(40)
Free
Number999
GLOBALENGINEERING
(9)
Free
ビューアー
GLOBALENGINEERING
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Description
"Region Select" is a new puzzle game for Android discovered by
Osaka City University Advanced Mathematical Institute using advanced mathematical knot theory.
Although its rules are as simple as Sudoku and Rubik's Cube, "Region Select" will challenge you to
think ahead like Japanese Go.
[Game explanation]
At the beginning there is a knot figure. The spaces between the lines are called "regions",
and the places where the lines meet are called "crossings".
MORE
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REGION SELECT
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+1
65
Tweet
ABOUT THIS APP
RATING:
(35)
UPDATED:
January 12, 2012
CURRENT VERSION:
1.0.8
REQUIRES ANDROID:
2.2 and up
CATEGORY:
Brain & Puzzle
INSTALLS:
10,000 - 50,000
last 30 days
SIZE:
7.1M
PRICE:
Free
CONTENT RATING:
Everyone

History of the game

2010. Region crossing change was defined by Kishimoto.

2010. S. showed that a region crossing change on knot diagrams is an unknotting operation.

2011. “Region Select ” and “Region Lighting” were created by Kawauchi, Kishimoto and S.

2011. “Region Select” was released to Android market!

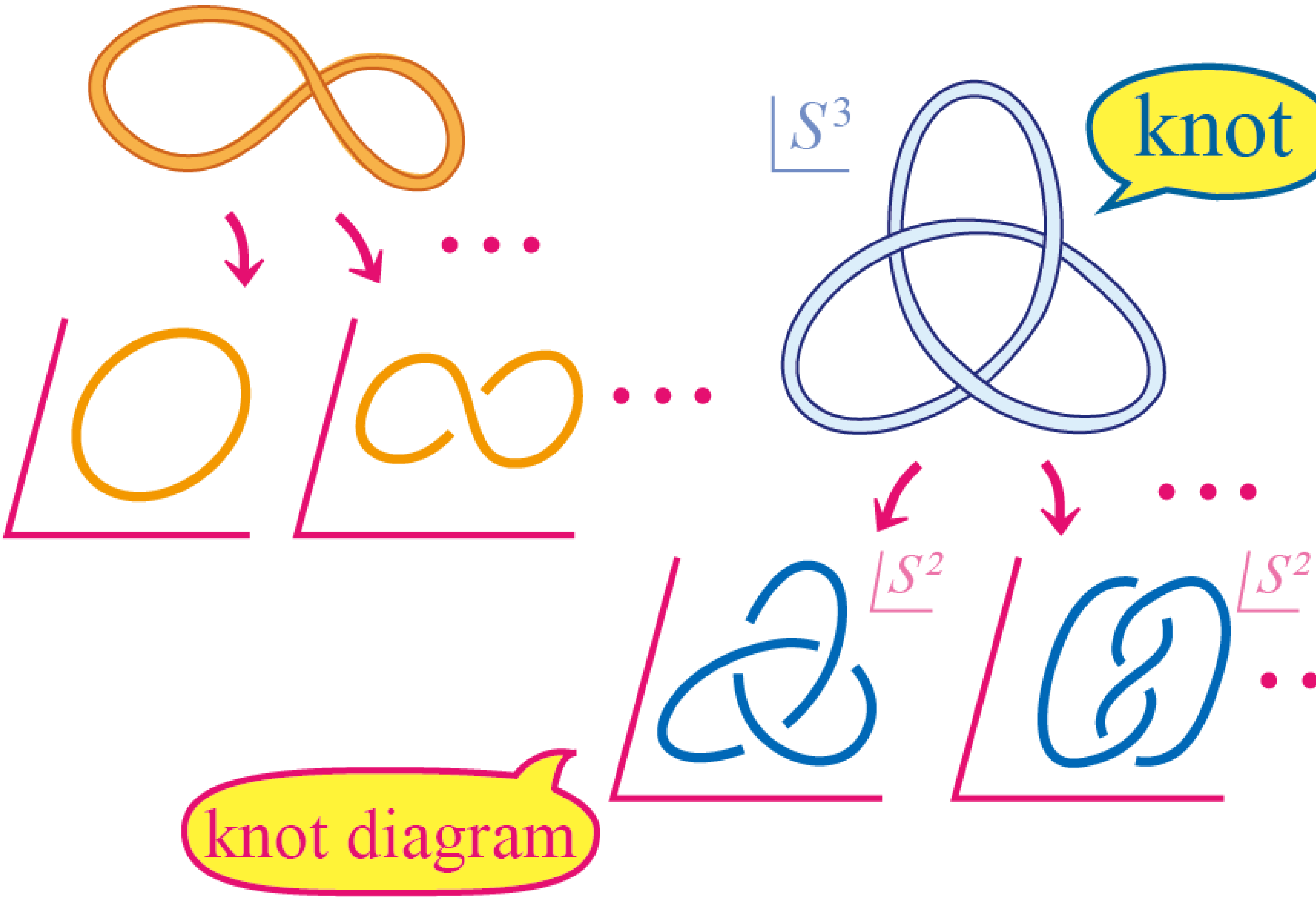
2012. A switching system was created by K-K-S.

Contents

§1. *Region crossing change*

- ***Region crossing change***
- ***Proof of the "key theorem"***

Knot

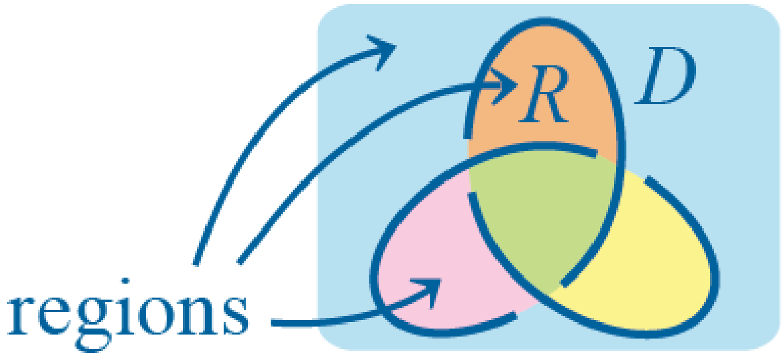


※ In this talk, knot diagrams are nontrivial and reduced.

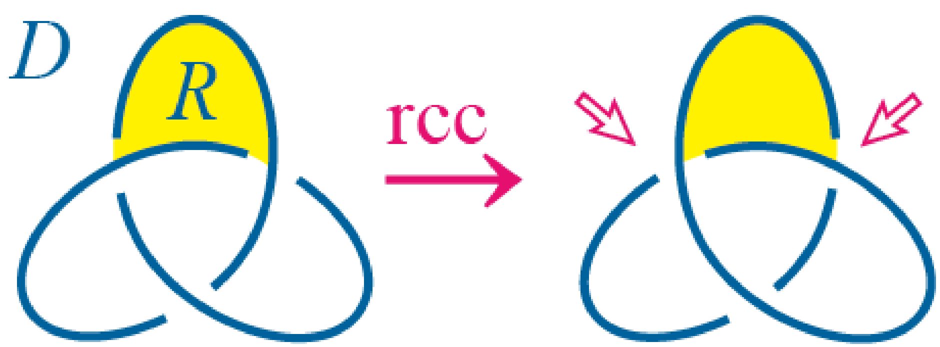
Region crossing change

D: a knot diagram

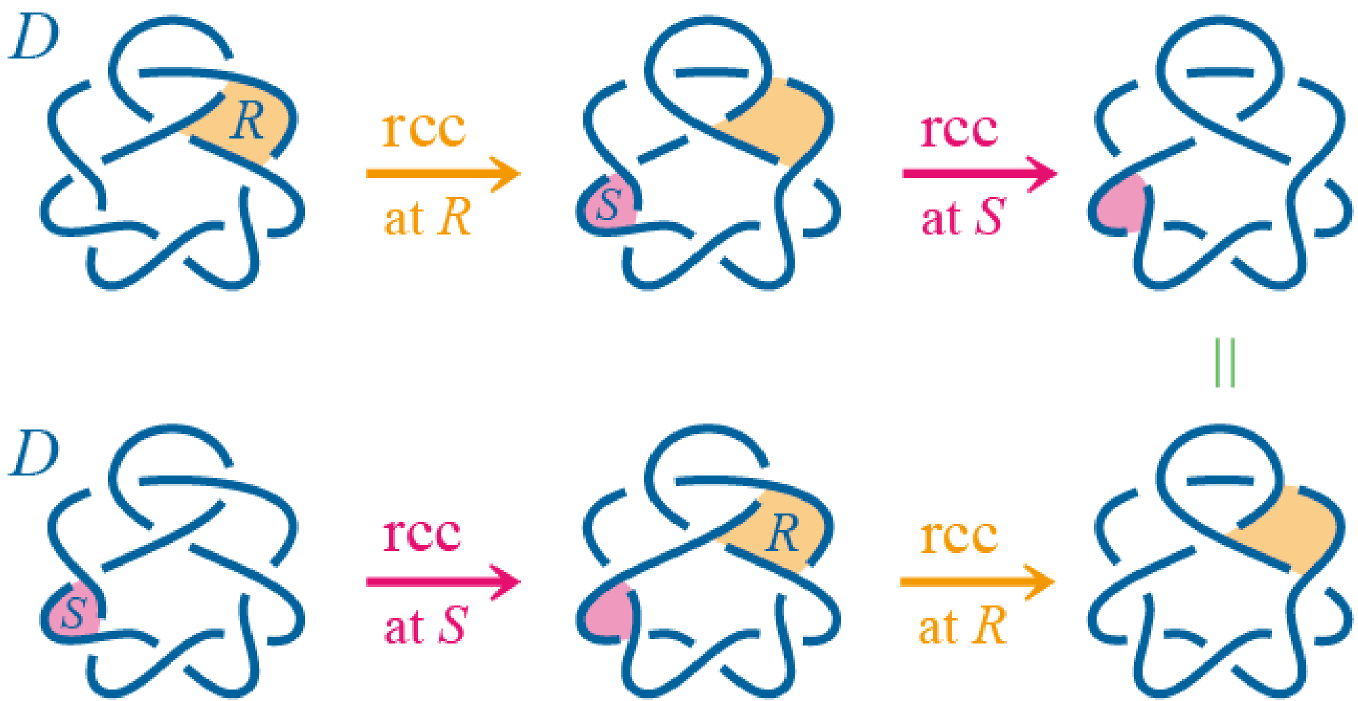
R: a region of D



A ***region crossing change*** at R is changing all of the crossings on ∂R.



Region crossing change

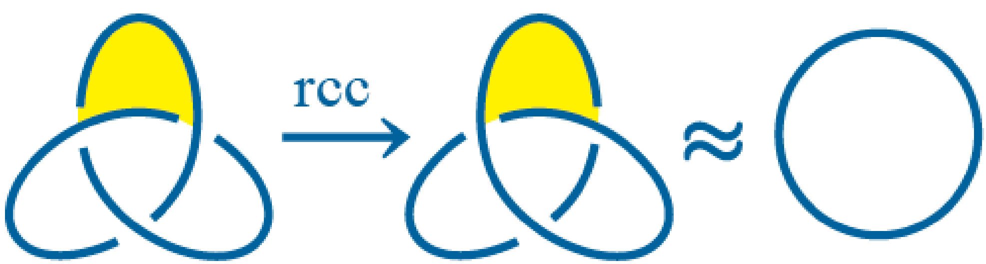


(Region crossing changes do not depend on the order.)

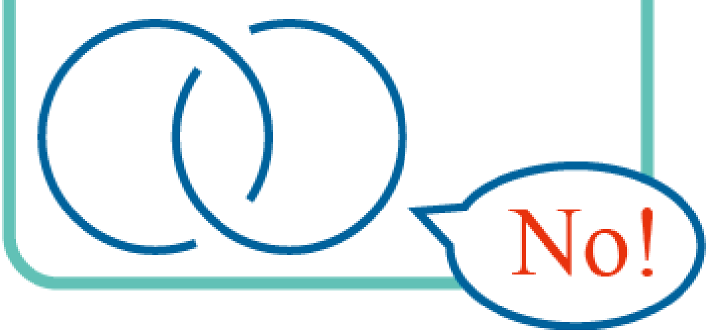
Kishimoto's Question

Kishimoto's question (2010)

Is a region crossing change on a knot diagram an <u>unknotting operation</u>?



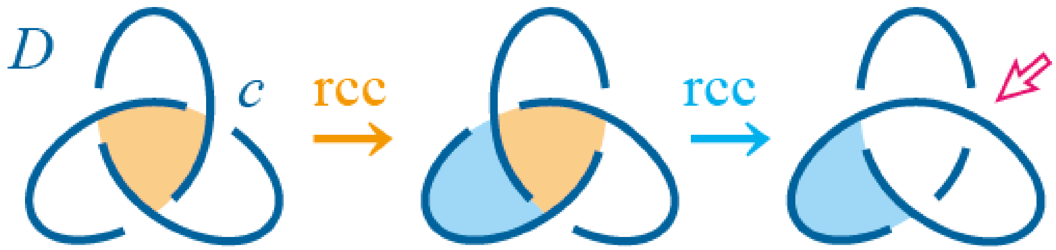
For links...



Key theorem

Key theorem (S. 2010)

We can change any crossing of a knot diagram by region crossing changes.

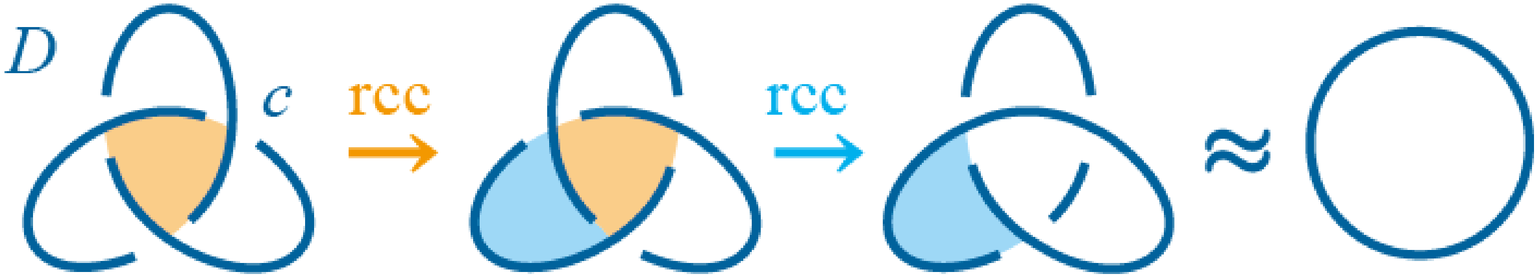


Reference

A. Shimizu, Region crossing change is an unknotting operation, arXiv: 1011.6304.

Key theorem

Corollary 1.
Region crossing change on a knot diagram is an unknotting operation.



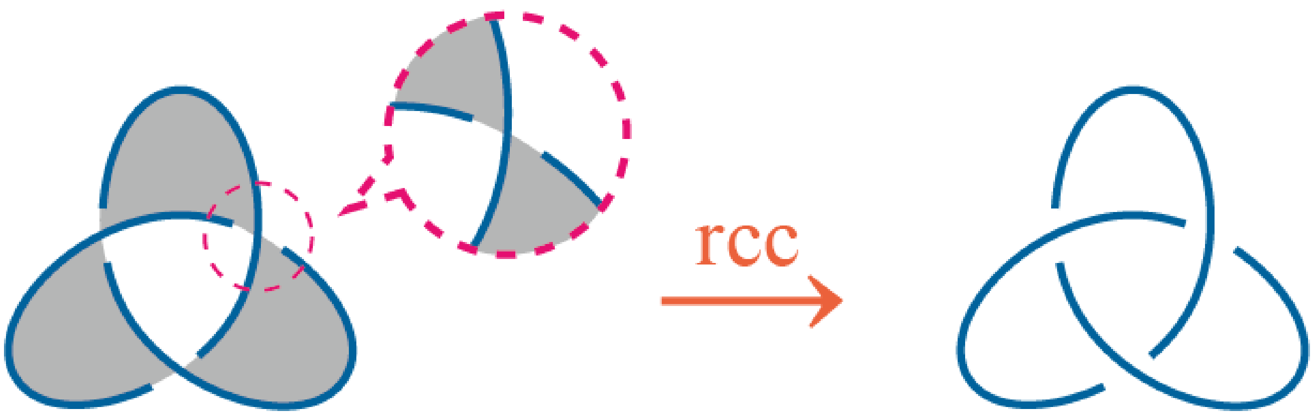
§1. *Region crossing change*

- *Region crossing change*
- *Proof of the key theorem*

Checkerboard coloring

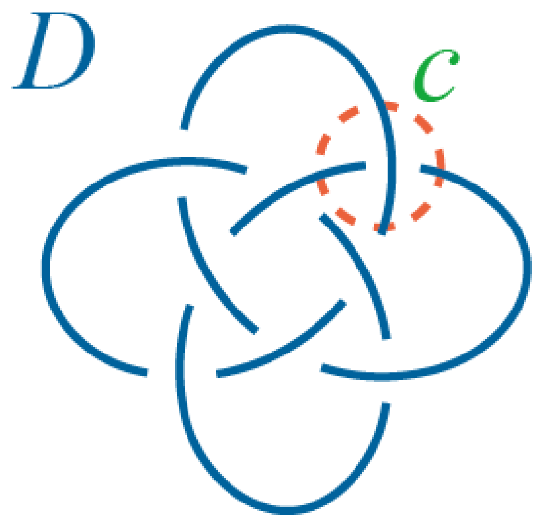
Proposition 2.
D : a knot diagram with a checkerboard coloring

$$D \xrightarrow[\text{black regions}]{\text{rcc at all the}} D$$



Proof of the key theorem

Proof.

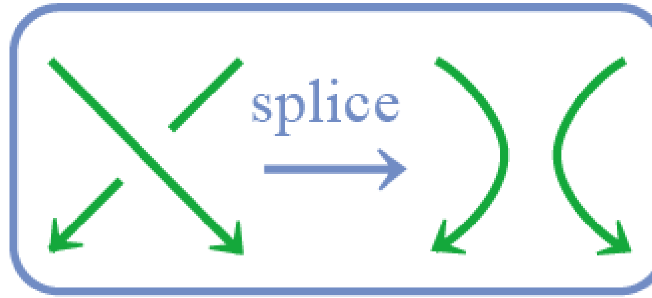


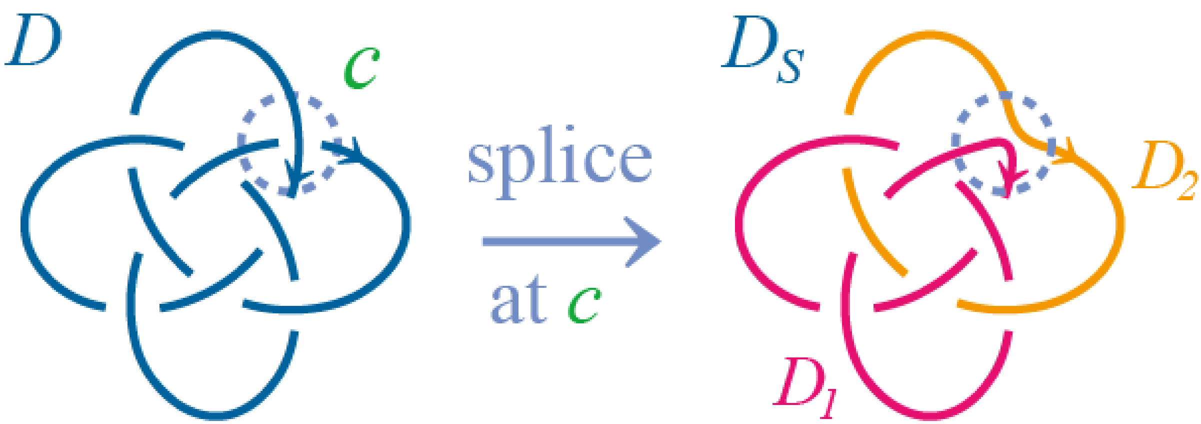
To obtain the regions s.t. we change c by the region crossing changes ...

Proof of the key theorem

Step 1.

Splice D at c.



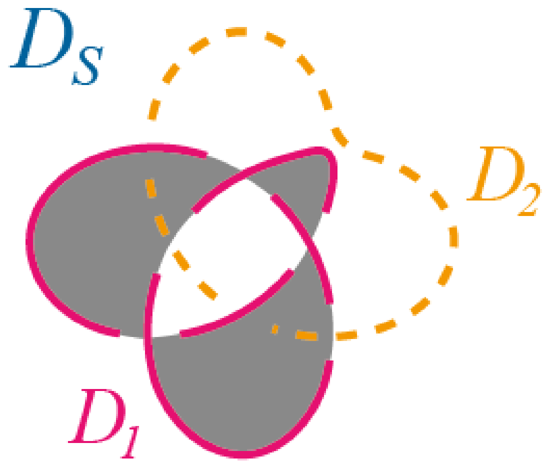


Then we obtain a two-component link diagram $D_S = D_1 \cup D_2$.

Proof of the key theorem

Step 2.

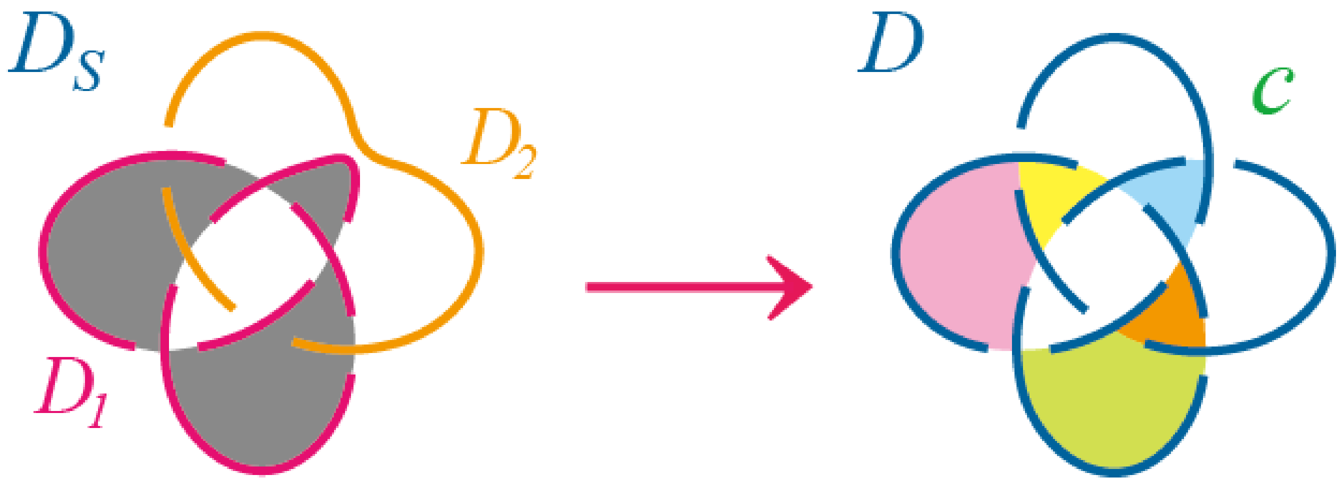
Apply a checkerboard coloring to only D_1.



Proof of the key theorem

Step 3.

Take the regions of D corresponding to the black-colored regions of D_S.



Thus, we obtain the regions. $\square$

§2. *Games (joint work with Kawauchi and Kishimoto)*

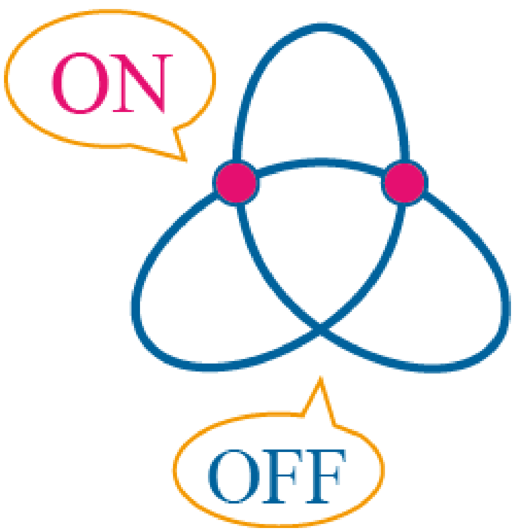
- *Region Select*
- *Region Lighting*

Region Select

D

a knot diagram

P

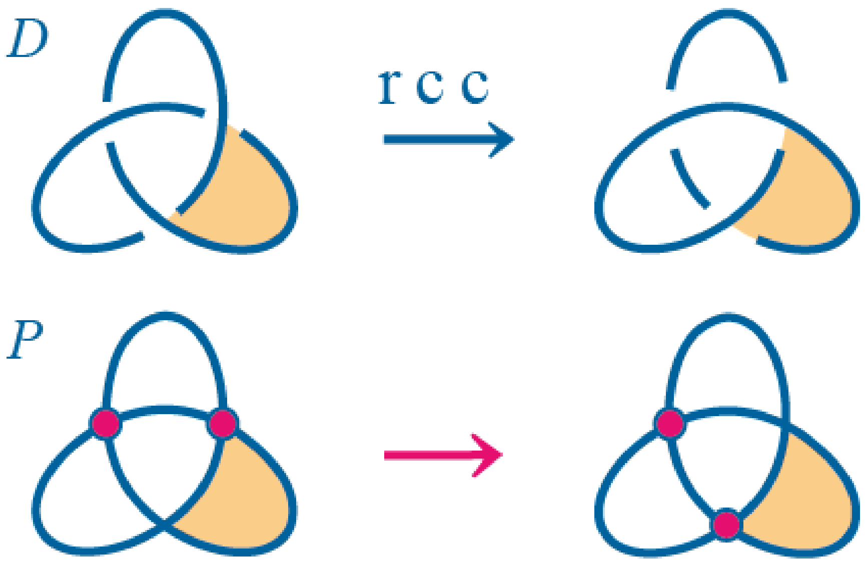


a knot projection

with lamps

Region Select

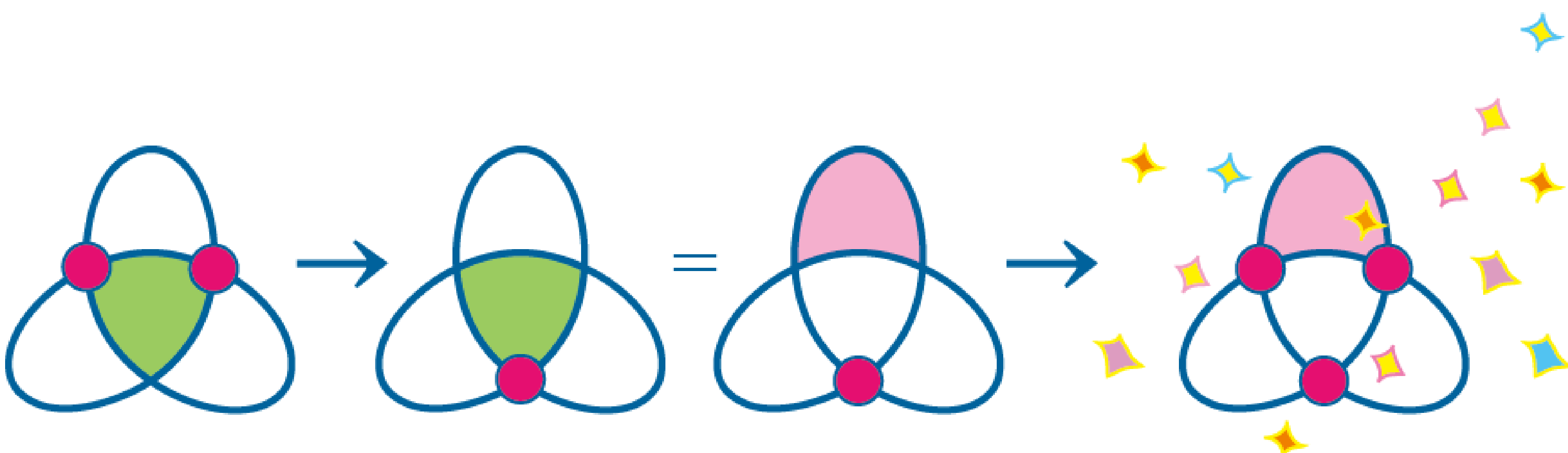
Region crossing change corresponds to...



changing ON/OFF at all of the lamps on the boudary of the region.

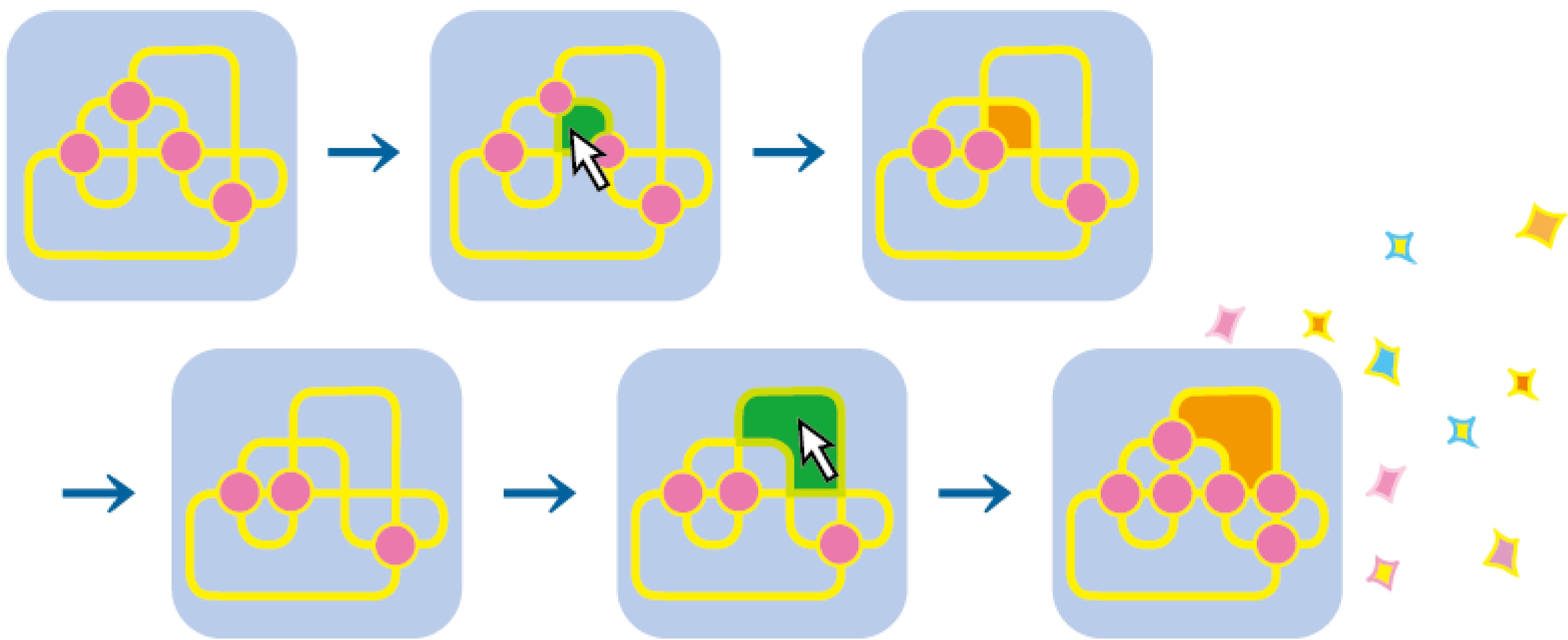
Region Select

By the key theorem, we can light up all of the lamps by region crossing changes for any knot projection with any state of lamps.



Region Select

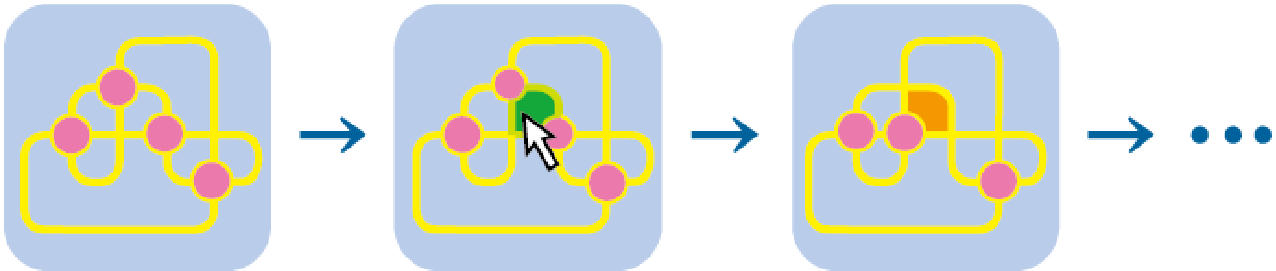
A Game Using Knot Theory



by Akio Kawauchi, Ayaka Shimizu and Kengo Kishimoto

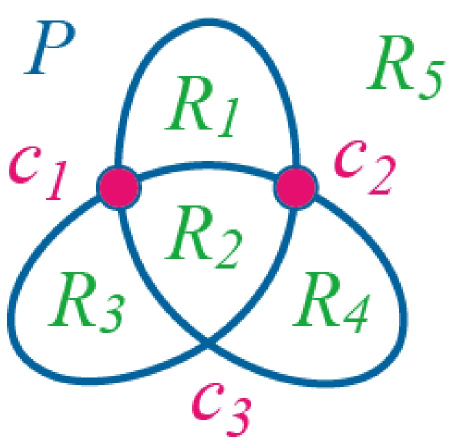
Japanese Patent Application (2011)

Region Select



- We can clear the game by $n/2+1$ or less clickings. (n: the number of crossings)
- For any region, we can clear the game without clicking the region.

How to find the regions (Taniyama and Kishimoto's method)



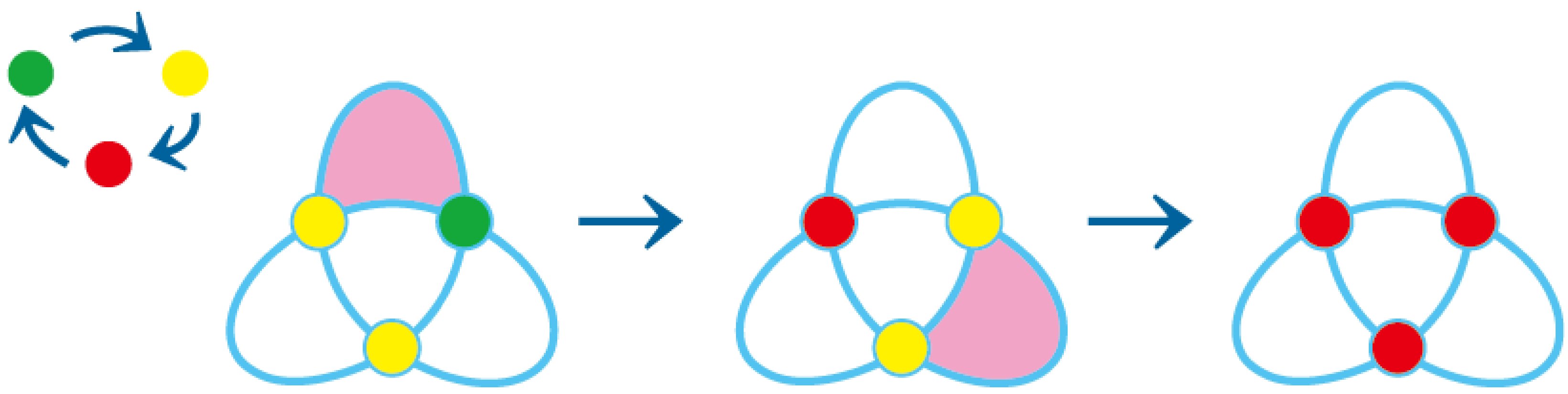
$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$(1+1=0)$$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Region Select w/ n colors (by Ahara and Suzuki)

We can play Region Select over mod n, too!

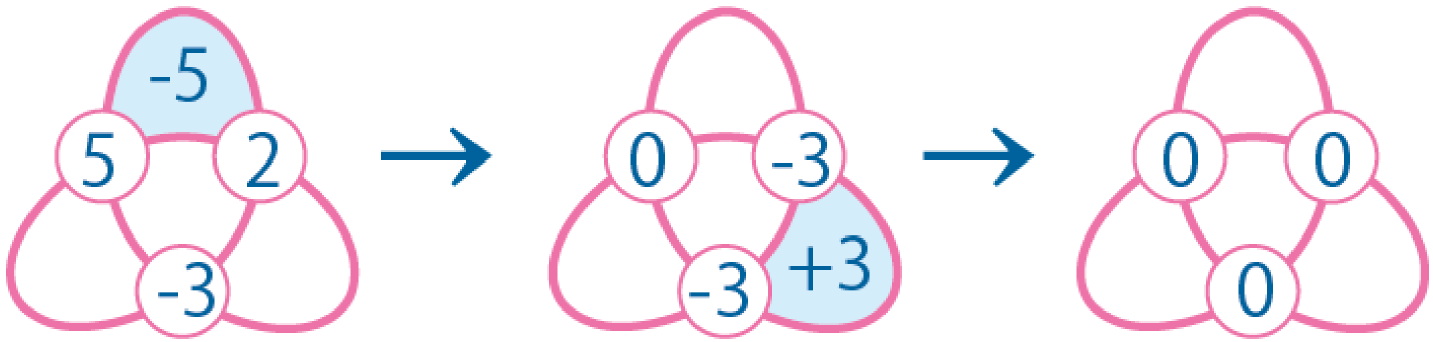


Reference

K. Ahara and M. Suzuki, An integral region choice problem on knot projection, arXiv, 1201.4539.

Region Select w/ numbers (by Ahara and Suzuki)

Moreover,
we can play Region Select over $\mathbb{Z}$!!



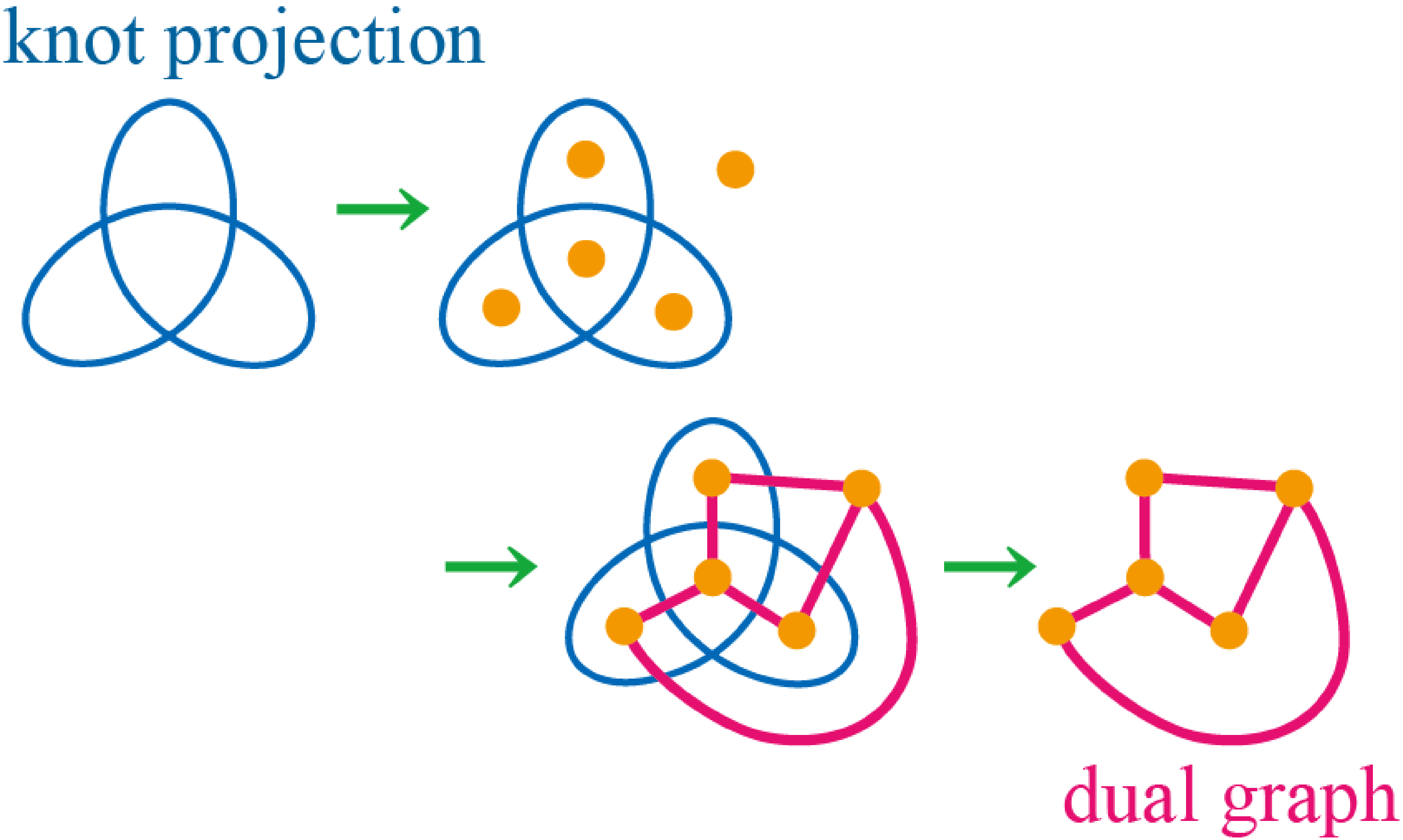
Reference

K. Ahara and M. Suzuki, An integral region choice problem on knot projection, arXiv, 1201.4539.

§2. *Games (joint work with Kawauchi and Kishimoto)*

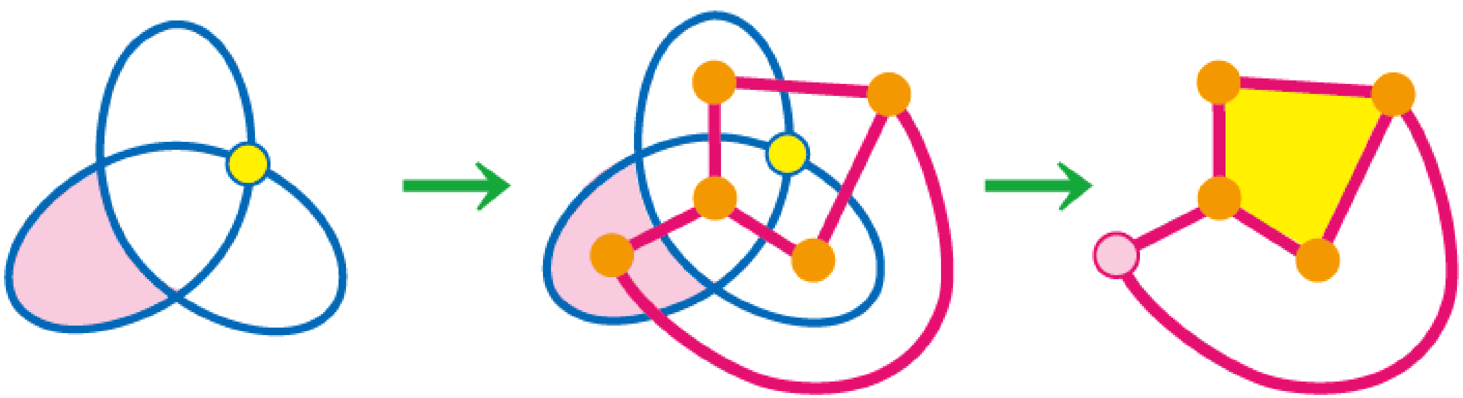
- ***Region Select***
- ***Region Lighting***

Region Lighting



Region Lighting

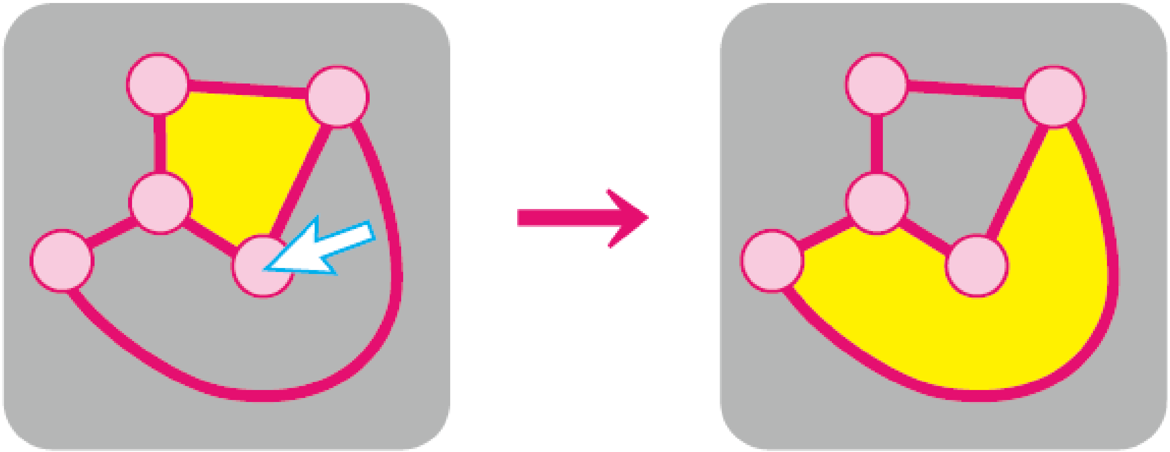
knot projection
with a lamp



dual graph

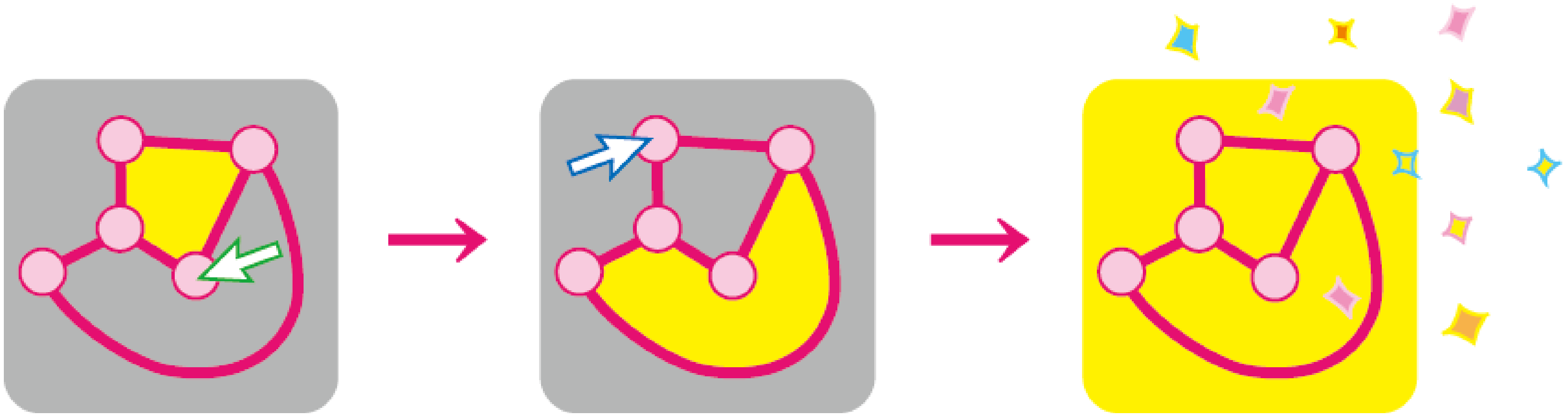
Region Lighting

Region crossing change corresponds to...



changing ON/OFF at all the regions around the vertex.

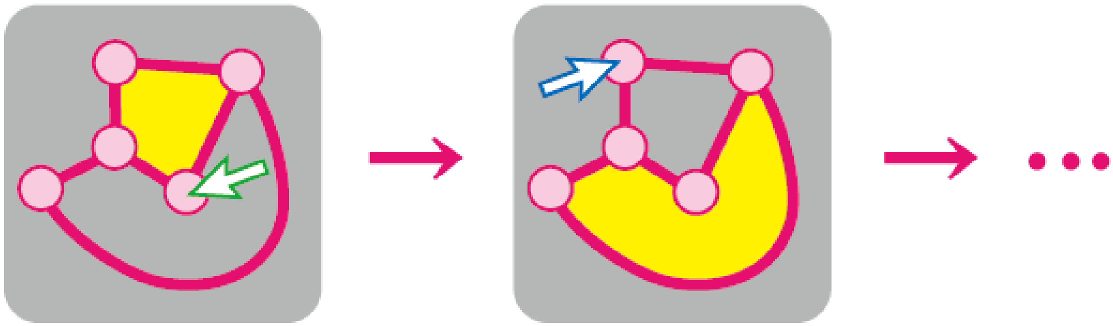
Region Lighting



by Akio Kawauchi, Ayaka Shimizu and Kengo Kishimoto

Japanese Patent Application (2011)

Region Lighting



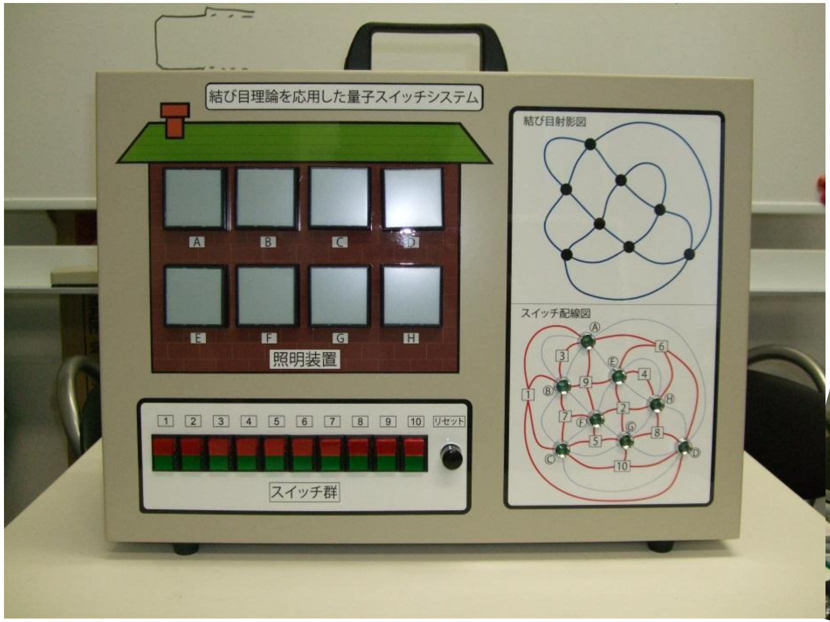
- We can clear the game by $v/2$ or less clickings. (v: the number of vertices)
- For any vertex, we can clear the game without clicking the vertex.

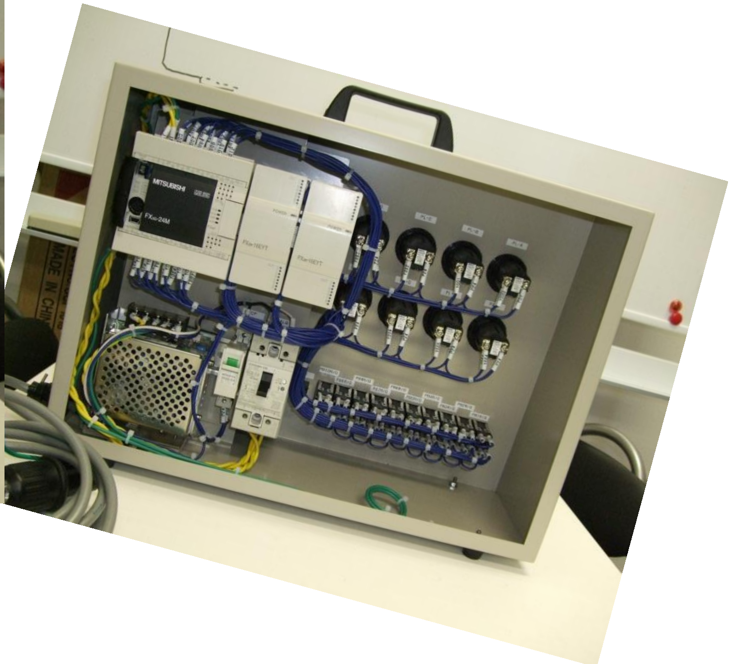
Future works

We will apply ***Region Select*** and ***Region Lighting*** to...

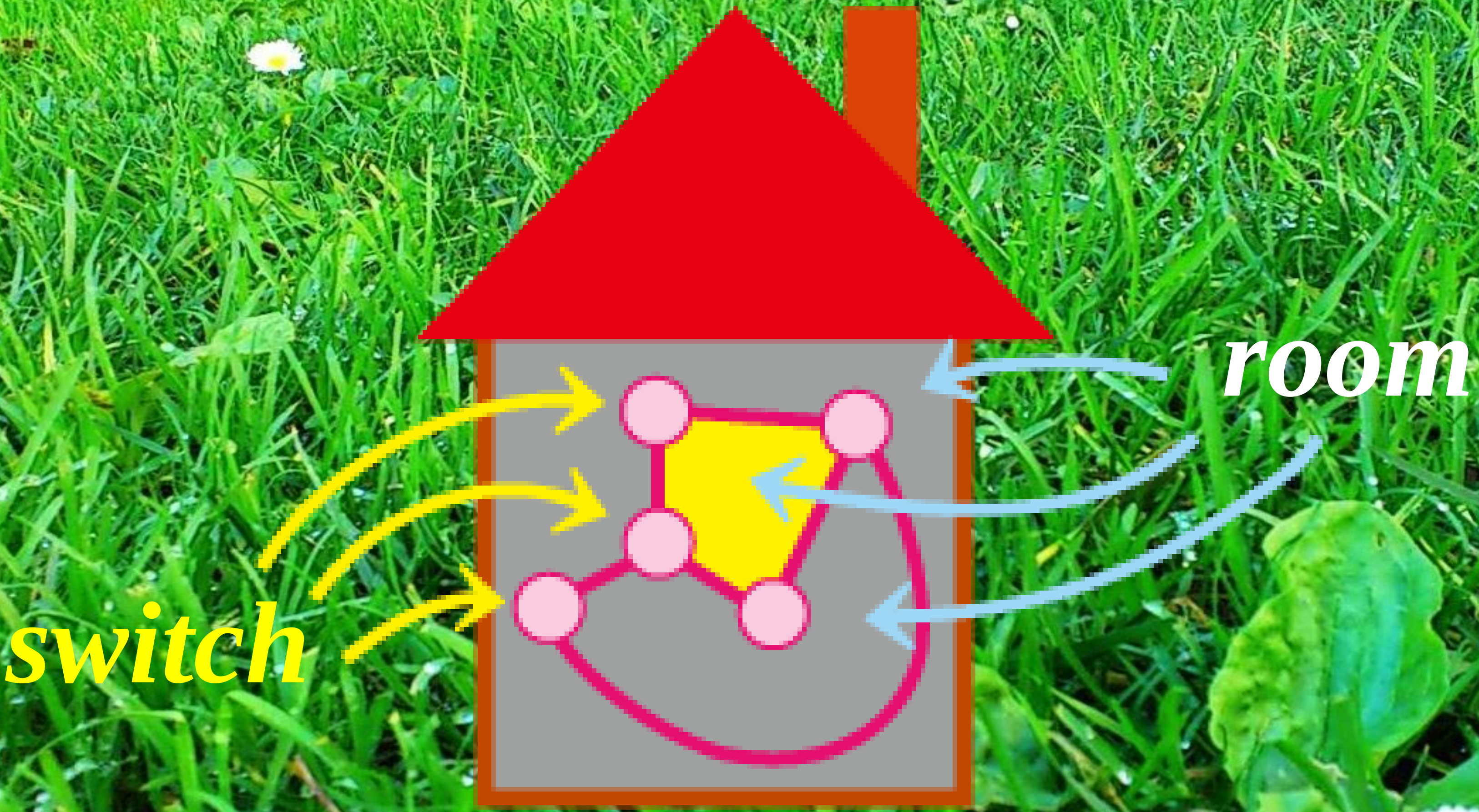
- primary education of graphics
- training cognitive functions during rehabilitation
- ⋮

§3. *Switching system*

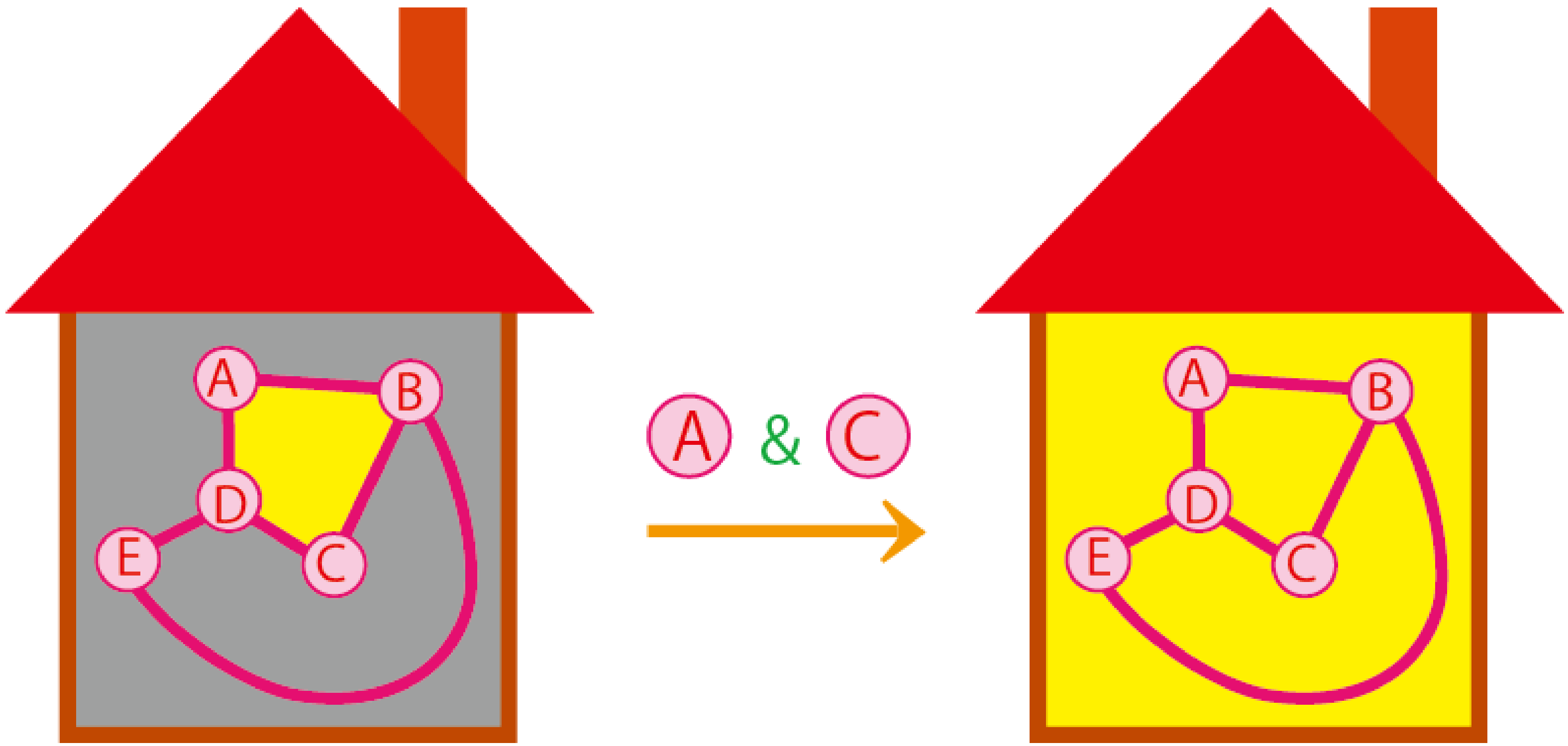




Switching system



Switching system



Switching system

- We can make any state of room lights by choosing $r/2+1$ or less switches. (r: the number of rooms)

- Even if one switch breaks down, this switching system works.


Thank you!